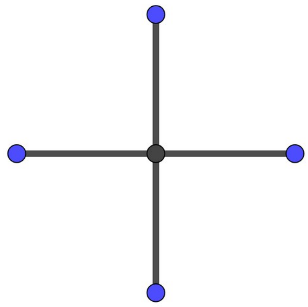
On intertwined polysigned p3 and equatorial geometry

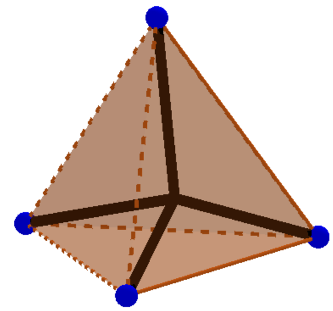
Precedent

The author of the polysigned numbers ゴールデン・P・ティム, in his web, presented the T3 tatrix as a possible way of combining the p2 and p3 symmetries for the three-dimensional space. In this brief article, other methodology is presented.

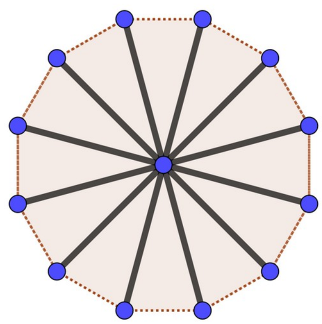
Equatorial considerations in the cuboctahedron

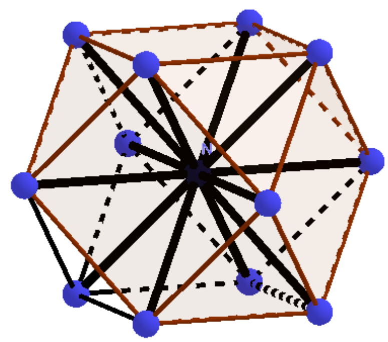
For the cuboctahedron twelve vertices, its cartesian coordinates the are given by **(±1,±1,0), (±1,0,±1), (0,±1,±1)**. The quadray coordinates[1] are given by the permutations of **(2,1,1,0)**.





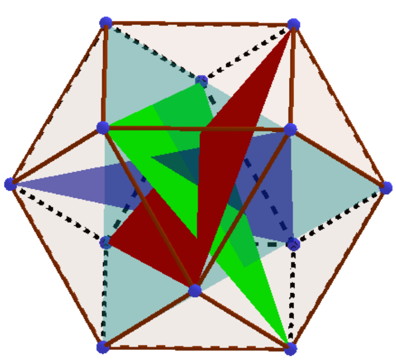
The antipodality of vertices is not preserved if we go from the square to the tetrahedron,

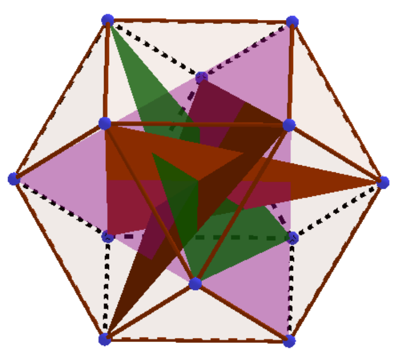


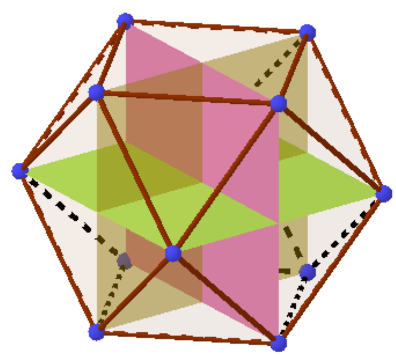


but, it may be preserved if we go from the dodecagon to the cuboctahedron.

The octahedron is member of the cross-polytope (hyperdiamond) family, and its dual is a parallelohedron. The dual of the cuboctahedron is also parallelohedron. If we consider the equatorial geometry, as opposed to geometry in the surface. We observe







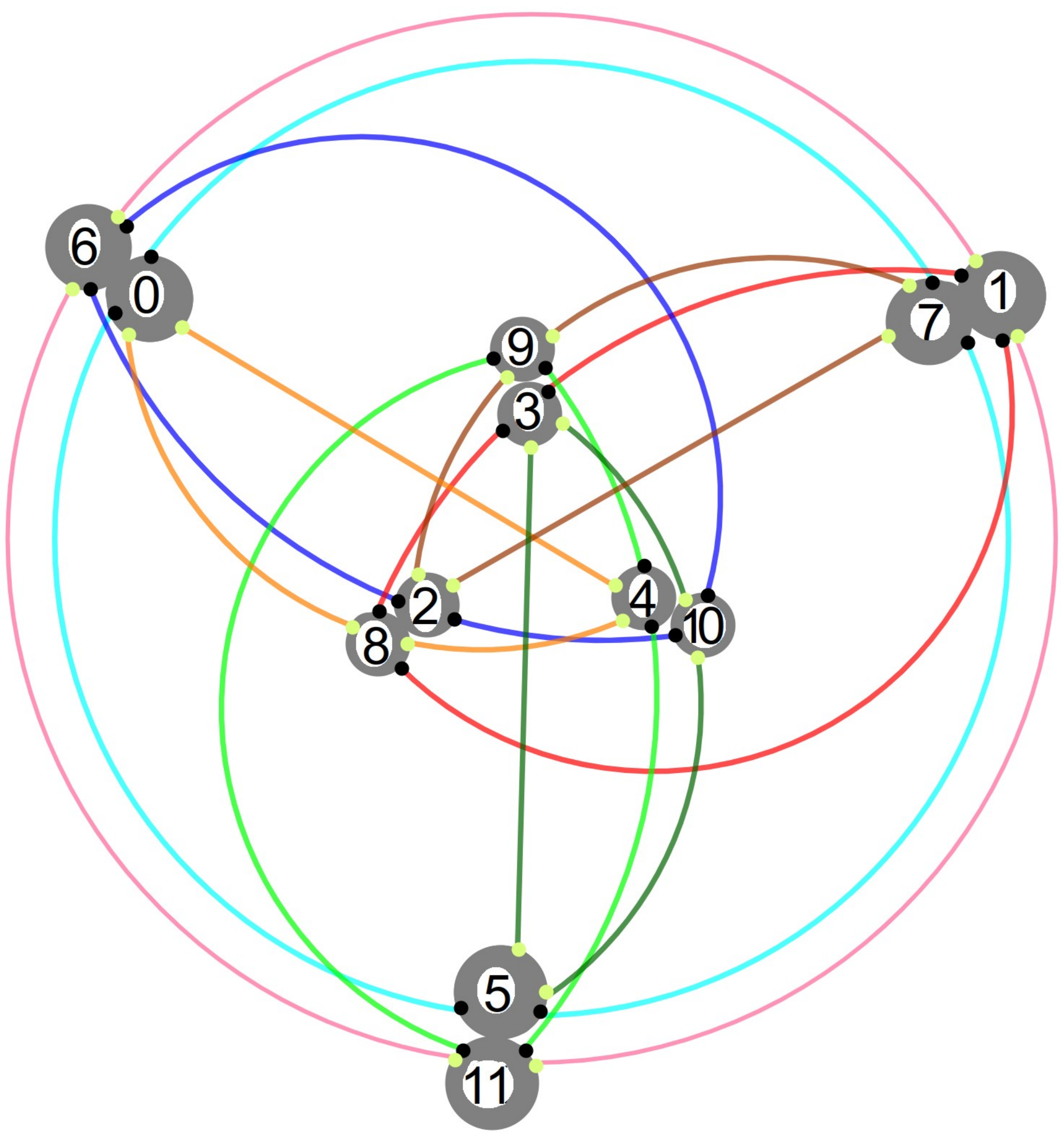
Also see [9].

Additive Conjoined P3 Twins or additive intertwined P3s

In p3, we have triple **(a,b,c)** and one equivalendo **(a,b,c) ~ (a+k,b+k,c+k)** .
The choice of a representative element may be **(a-l,b-l,c-l) with l being min(a,b,c)** .

In T12 system, we have the duodecuple **(a,b,c,d,e,f,g,h,i,j,k,l)**
Also can be used $e_0a + e_1b + e_2c + e_3d + e_4f + e_5g + e_6h + e_7i + e_8j + e_9k + e_1l$
or **一b ＋c ＊d ＃e 生f 兆g 花h 幸i 逃j 恋k 渓l 業a**

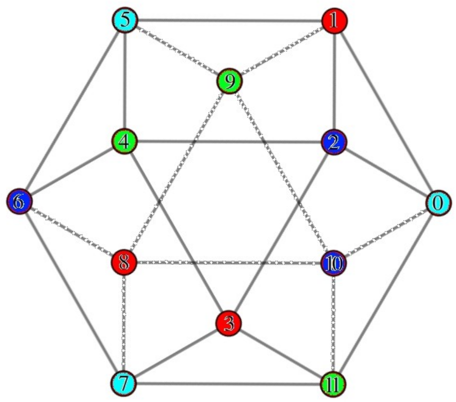
The T12 system does not have one, but eight equivalendos $\sim^1$ $\sim^2$ $\sim^3$ $\sim^4$ $\approx^1$ $\approx^2$ $\approx^3$ $\approx^4$,
where each one belongs to the family ~ (black), or to the family ≈ (yellow),
each one working for one of the eight identified subtriples **(.. ,a, .. ,b, .. ,c, ..)**
of the duodecuple **(⓪,①,②,③,④,⑤,⑥,⑦,⑧,⑨,⑩,⑪)**

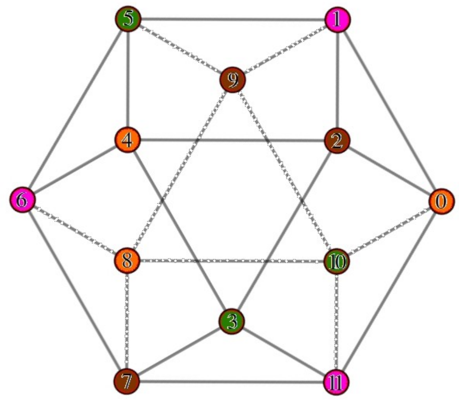


[Göldenigsberg graph]

$\sim^1$ works for rays ②⑥⑩	**$\approx^1$ works for rays ②⑨⑦**
$\sim^2$ works for rays ⓪⑦⑤	**$\approx^2$ works for rays ③⑩⑤**
$\sim^3$ works for rays ④⑪⑨	**$\approx^3$ works for rays ⓪④⑧**
$\sim^4$ works for rays ①⑧③	**$\approx^4$ works for rays ①⑪⑥**

Like a couple stages in a round-robin tournament[2]
(.. ,a, .. ,b, .. ,c, ..) <--> (.. ,a+k, .. ,b+k, .. ,c+k, ..)





The vextex ⓪ makes subtriple with ⑦ ⑤ , but also, with ④ ⑧.
In other words, it is able to cancellate with ⑦ ⑤, an also with ④ ⑧.
This is not a problem as we can observe.

With the equivalendo $\sim^2$ dedicated to the rays ⓪,⑦,⑤
(10,0,0,0,0,4,0,4,0,0,0,0) $\sim^2$ **(6,0,0,0,0,0,0,0,0,0,0,0)**

with the equivalendo $\approx^3$ dedicated to the rays ⓪,④ **and** ⑧
(10,0,0,0,1,0,0,0,1,0,0,0) $\approx^3$ **(9,0,0,0,0,0,0,0,0,0,0,0)**

Since all equivalendos are mutually compatible, do not produce algebraic atrocities for entry-wise addition, one may apply the equivalendo $\sim^2$ first, and after, the equivalendo $\approx^3$
(10,0,0,0,1,4,0,4,1,0,0,0) $\sim^2 o \approx^3$ **(5,0,0,0,0,0,0,0,0,0,0,0)**

or in the reverse order
(10,0,0,0,1,4,0,4,1,0,0,0) $\approx^3 o \sim^2$ **(5,0,0,0,0,0,0,0,0,0,0,0)**
and will be referencing to the same point in three-dimensional space.

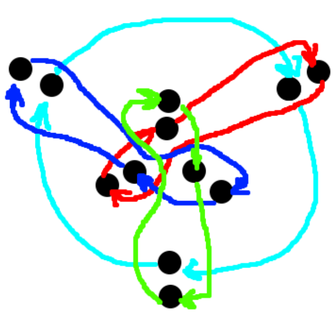
From now on, a choice of use the symbol **=** instead of the equivalendos can be made.
(.. ,a, .. ,b, .. ,c, ..) = (.. ,a+k, .. ,b+k, .. ,c+k, ..)

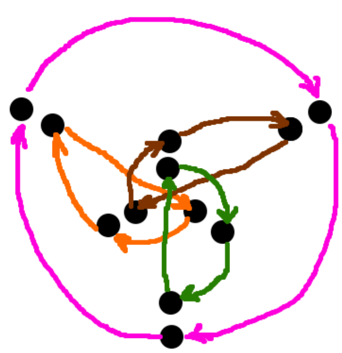
An strange multiplicative

Some example of simple products for structure of twelve elements using clock arithmetic
a + b ≡ c (mod 12) for {0,1,2,3,4,5,6,7,8,9,10,11}
a - b ≡ c (mod 12) for {0,1,2,3,4,5,6,7,8,9,10,11}
a · b ≡ c (mod 13) for {1,2,3,4,5,6,7,8,9,10,11,12}
x ^ y ≡ c (mod 13) for {1,2,3,4,5,6,7,8,9,10,11,12}

Also, the product can be isolated from the dynamics of the cuboctahedron vertices of some three-dimensional algebraic system, like p4, k4, ℂ4 or ¤ product. See [2] and [8].

Now, as opposed to the simple framework of the of the tetrahedrical quadruple, the cuboctahedral duodecuple present an additional observation.





For example, a term with index ⓪ is able to make p3 rotations towards ⑦ ⑤, but, also is able to do p3 rotations to ④ ⑧. We have some options of how to proceed. The rest of the section is centered in how to realise into the product the dynamics of the above figure. Notice it has clockwise oriented edges. In addition, how one labels the vertices of the cuboctahedron is an issue, and may also, help or complicate the study.

I. Two products, one label per ray

term := $\mathbf{e^{i}m}$

We choose two products.
To perform the first product, we use parentheses **( )**
$\mathbf{(e^{i}m_1)(e^{j}m_2) = (e^{i}e^{j})m_1m_2}$
To perform the second product, we use square brackets **[ ]**
$\mathbf{[e^{i}m_1][e^{j}m_2] = [e^{i}e^{j}]m_1m_2}$

II. One product, two labels per ray (term with conjoined indexes)

We label each ray with two indexes from 0 to 11

term := $\mathbf{e^{i\_j}m}$

To perform the product with the left index, we use parentheses **( )**
$\mathbf{(e^{i\_j}m_1)(e^{k\_h}m_2) = (e^{i\_j}e^{k\_h})m_1m_2}$
To perform the product with the right index, we use square brackets **[ ]**
$\mathbf{[e^{i\_j}m_1][e^{k\_h}m_2] = [e^{i\_j}e^{k\_h}]m_1m_2}$

III. Hybridize several products into one

	a	**b**	**c**	**d**
a	**{aa}**	**(ab)**	**(ac)**	**(ad)**
b	**[ba]**	**{bb}**	**(bc)**	**(bd)**
c	**[ca]**	**[cb]**	**{cc}**	**(cd)**
d	**[da]**	**[db]**	**[dc]**	**{dd}**

For example, the quaternion product can be viewed as an appealing hydrid of simpler products

IIII. Additive integrity of terms **A** $\underset{\sim}{+}$ **B**

(termA_termB)(termC_termD) = (termA)(termC)_(termB)(termD)

Also see [2],[3],[5],[6],[7] and [8].

V. Forget about trying of find a product(s) compatible with the multiplicative conjoined p3 dynamics, and instead, find a product with some attractive feature, like a product of arity 3 without identity elements, or a product that satisfy **|a||b| ⇒ |ab|** (some sort of very distant relative of a field, but with not-too-nice algebraic properties)

The additives and multiplicative dynamics associated with the dodecahedron and the icosidodecahedron are more complicated. In the four-dimensional case, for the 24-cell vertices, the additive dynamics are even more complicated, but the product do not need to be found because the icosian group (under the quaternion product) for the 24-cell can be used to extract the multiplication table.

Of course, one of the attractive features that has the composition property **|a||b|=|ab|** is its associated sense of closeness. For the case of polar format of complex numbers, the multiplication is $\mathbf{(r1,\alpha1)(r2,\alpha2) = (r1r2,\alpha1+\alpha2)}$, that emanates visually. More formalisms of rotation in three-dimensional space, rigid body orientation mnemonics, angle addition, composition of rotation to be found ?

Yaw, pitch, and roll...

Resources

1. Kirby Urner (October 7, 1997 / March 25, 2000), http://www.grunch.net/synergetics/quadintro.html
2. Sci.math Newsletter (May 28, 2006 / Jun 25, 2006) thread "Unity and its interpretation" https://groups.google.com/g/sci.math/c/AKO-6IRsH58
3. V. Abramov , R. Kerner , O. Liivapuu (June 19, 2020), "Algebras with ternary law of composition combining Z2 and Z3 gradings"
5. Redouane Bouhennache (june 27 2016), "Truly hypecomplex numbers : Unification of numbers and vectors"
6. John Shuster, Jens Koplinger (august 15 2010), "Elliptic complex numbers with dual multiplication"
7. S. Egi (september 2, 2018), "Scalar and Tensor Parameters for Importing Tensor Index Notation including Einstein Summation Notation"
8. Tanaka (july 2021) "Pacman Product for Polysigned numbers"
9. 1ciekaw (Jan 6, 2018), "Four interlocking triangles" https://www.youtube.com/watch?v=OT6NmF9cAiM
10. Florian Cajori (1928 and 1929), "A history of mathematical notations" Volume I and II
11. Timothy Golden Homepage, http://www.bandtechnology.com/

バッタ たなか signing off 。
September 3, + @ + +

// Thanks to Tomruen

// courtesy of ゴールデン・P・ティム comment, Heavyside can be found in the cover of 'In Praise of Simple Physics' by Paul Nahin

// Round-robin comment, a reference provided by Ross A. Finlayson

// Vortex cells of "Physical Lines"

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